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THEORETICAL BUCKLING LOADS OF BORON/ALUMINUM AND GRAPHITE/RESIN FIBER-COMPOSITE ANISOTROPIC PLATES

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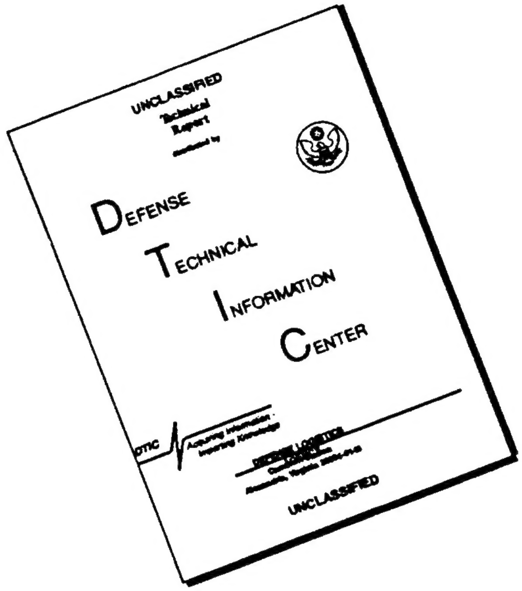

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16. Abstract

Theoretical results are presented for the buckling of anisotropic plates. The plates are subjected to simple and combined in-plane loading. The plates are made from fiber composite material of boron/aluminum or high-modulus graphite/resin. The results are presented in nondimensional form as buckling load against fiber orientation angle for various plate aspect ratios. The results indicate that buckling loads of boron/aluminum plates are independent of fiber direction if the plate aspect ratios are greater than about 1, and moderately dependent when this ratio is less than about 1. In addition, the results indicate that the buckling loads are independent of aspect ratio for plates with aspect ratios greater than about 2. Boron/aluminum composite plates can resist buckling loads more efficiently than graphite/resin composites on a specific buckling stress basis. The numerical algorithm and a listing of the computer code used to obtain the results are included.

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THEORETICAL BUCKLING LOADS OF BORON/ALUMINUM AND GRAPHITE/RESIN FIBER-COMPOSITE ANISOTROPIC PLATES

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SUMMARY

Theoretical results are presented for designing with boron/aluminum composites when these composites are buckling critical. The composites are assumed to be rectangular plates with four simply supported edges. They are subjected to single and combined normal and shear loads in the plane of the plate. The plates are made from a unidirectional composite whose fiber direction is oriented at some angle to the load direction.

The design data are presented in nondimensional form as buckling load against orientation angle for several plate aspect ratios. The results indicate that the buckling loads of boron/aluminum plates are independent of fiber orientation if the plate aspect ratio is greater than approximately 1. The buckling loads are moderately dependent on the orientation angle for plates with aspect ratios less than about 1. The buckling load is independent of aspect ratio in plates with aspect ratios greater than about 2.

Comparison of buckling results for boron/aluminum composite plates and Thornel-75/epoxy composite plates indicates that the boron/aluminum composite plates resist buckling loads more efficiently than the Thornel-75/epoxy composite on the basis of specific buckling strength. The results also indicate that the buckling loads of boron/aluminum composite plates can be predicted using orthotropic theory if their aspect ratio is greater than about 1.

The numerical algorithm used to solve the buckling problem and listing of the corresponding computer code through which the results were obtained are included.

INTRODUCTION

Feasibility studies for the space shuttle indicate that the use of advanced fiber composite structural components can result in a considerable increase in payload in the shuttle system. Boron/aluminum and graphite/resin fiber composites are leading contenders for shuttle applications because these composites offer high stiffness-to-density

and high strength-to-density ratios. Panels made from these materials will have to meet both material strength and buckling requirements. This report deals with a theoretical investigation of the buckling of flat rectangular panels made from boron/aluminum and Thornel-75/epoxy fiber composites.

Several papers have appeared recently dealing with the buckling of anisotropic plates (refs. 1 to 8). However, design data are not available for flat panels subjected to compressive loads and made from advanced fiber composites such as boron/aluminum and graphite/resin. The method described in reference 3 has proved efficient in buckling studies of boron/epoxy plates. This method is used herein to generate design data for boron/aluminum plates. Some data for Thornel-75 graphite/epoxy resin plates are also generated for comparison purposes. Data were generated for aspect ratios of 1/2, 1, 2, and 4.

The panels considered are anisotropic and simply supported. They are subjected to combined in-plane (normal and shear) load (fig. 1). The material is a unidirectional composite with the fiber direction oriented at an arbitrary angle to the load direction (fig. 1). The analytical algorithm used is the assumed mode method in conjunction with the Galerkin method. A computer code was developed based on the Galerkin method, and the code was used to generate the theoretical design data presented herein. A brief description of the analytical method is given in the report. All symbols are defined in appendix A. The numerical algorithm used to solve the resulting eigenvalue problem is described in appendix B. Input data sample sheets with explanations are given in appendix C. A listing of the computer program with sample cases is given in appendix D.

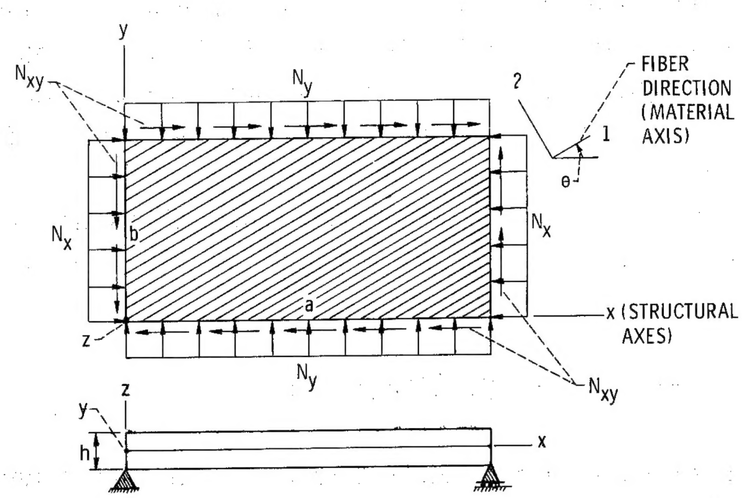


Figure 1. - Fiber-composite plate geometry and loading - all four edges simply supported (aspect ratio = a/b).

BRIEF DESCRIPTION OF UNDERLYING THEORY

The underlying theory for buckling loads of anisotropic plates is described in reference 3 with pertinent discussions in references 6 to 8. Briefly, this theory consists of expressing the potential energy of a plate in terms of displacement variables. Taking the variation of the potential energy function yields the field equation and the corresponding boundary conditions. The resulting system then is solved by the assumed mode technique in conjunction with the Galerkin method.

The equation resulting after the variation of the energy function is

$$\int_0^a \int_0^b \Big[D_{11}w,_{xxxx} + 2(D_{12} + 2D_{33})w,_{xxyy} + D_{22}w,_{yyyy} + 4D_{13}w,_{xyxx} + 4D_{23}w,_{xyyy} - \left(\bar{N}_x w,_{xx} + 2\bar{N}_{xy}w,_{xy} + \bar{N}_y w,_{yy}\right)\Big] \delta w \, dy \, dx + \int_0^b \left(D_{11}w,_{xx} + D_{12}w,_{yy} + 2D_{13}w,_{xy}\right)\Big|_0^a \delta w,_x \, dy + \int_0^a \left(D_{21}w,_{xx} + D_{22}w,_{yy} + 2D_{23}w,_{xy}\right)\Big|_0^b \delta w,_y \, dx = 0 \qquad (1)$$

(The notation is defined in appendix A.) The area integral represents the field equation, and the line integrals represent the boundary conditions.

The assumed buckling mode described in reference 3 is represented by a Fourier double sine series. This mode satisfies the imposed boundary conditions, but it does not satisfy the natural boundary conditions if the material and structural axes do not coincide. However, the mode is forced to satisfy the natural boundary conditions approximately through the Galerkin method as discussed in reference 3.

Substituting the assumed mode in equation (1), applying the Galerkin method, and carrying out the algebra result in a set of linear equations which represent the eigenvalue problem of the plate. This system is coupled for either a combination of shear and normal loads and/or noncoincident material and structural axes.

The eigenvalue problem is solved by using the Power method, which is a highly effective iterative numerical technique in seeking the largest eigenvalue of the system. The indicial equations which were used to generate this system and the Power method are given in appendix B in outline form.

BRIEF DESCRIPTION OF COMPUTER PROGRAM

The numerical algorithm described in appendix B has been transformed into a computer code which is rather simple and can be generated from the information supplied in appendix B. A FORTRAN IV compiled listing of a computer program based on the algorithm in appendix B is given in appendix D, with sample cases and output. Input data sample sheets are given in appendix C.

The inputs to the code are composite system identification, fiber volume ratio, orientation angle, plate aspect ratio, and flexural rigidities. The outputs are the number of terms in the assumed mode series expansion required for convergence, the relative error between the last two iteration cycles, the buckling load, and topo-plot data of the buckled shape of the plate normalized with respect to the largest deflection.

The algorithm described in appendix B runs into difficulty when the shear buckling load of a plate is sought. In this particular case, the difficulty is bypassed by including normal loads which are a very small fraction of the shear load. Further discussion on why these difficulties arise is presented in reference 3.

THEORETICAL DESIGN DATA

The theoretical design data generated herein are based on the schematic illustrated in figure 1. In this figure, the type of loading condition, the plate geometry, and the fiber orientation are defined. The x-y coordinate reference system is referred to as the structural axes system. The fiber direction coordinate system which is located at the angle θ from the structural axes system is referred to as the material axes system. The loading conditions are identified by N_x, N_y, and N_{xy} as is noted in the figure.

The flexural stiffnesses required in calculating the buckling loads were calculated by using the computer code described in reference 9. Typical values of the elastic constants of the plate along its material axes are given in table I for boron/aluminum and Thornel-75/epoxy composites with a fiber volume ratio of 0.5. The flexural rigidities are computed as functions of the orientation angle using the data in table I.

TABLE I. - THEORETICAL UNIDIRECTIONAL COMPOSITE

PROPERTIES AT FIBER VOLUME RATIO OF 0.5

Property	Boron/aluminum	Thornel-75/epoxy
Longitudinal modulus, N/cm^2 (psi)	24.2×10^6 (35.0×10^6)	26.0×10^6 (37.8×10^6)
Transverse modulus, N/cm^2 (psi)	16.8×10^6 (24.3×10^6)	6.9×10^6 (1.0×10^6)
Shear modulus, N/cm^2 (psi)	8.0×10^6 (11.6×10^6)	0.44×10^6 (0.63×10^6)
Poisson's ratio	0.24	0.25
Density, g/cm^3 ($lb/in.^3$)	2.62 (0.095)	1.55 (0.056)

Buckling of Boron/Aluminum and Thornel-75/Epoxy Panels

Buckling loads for a single loading condition for panels made of boron/aluminum and Thornel-75/epoxy are illustrated in figure 2, where the specific buckling stress has been plotted as a function of the orientation angle for an aspect ratio of 2. The schematic in the figure illustrates the type of load condition as well as the orientation angle. As can be seen, in this figure, boron/aluminum composites are more efficiently utilized than

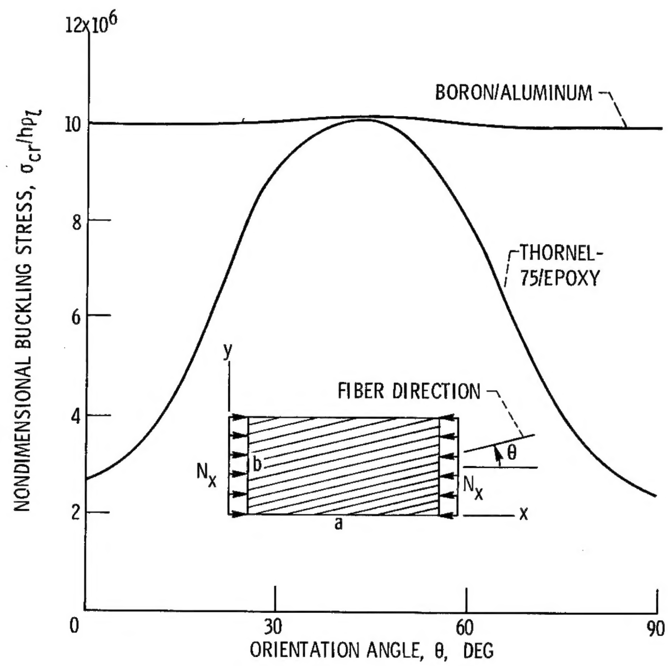


Figure 2. - Specific buckling stress of two fiber-composite plates - all four edges simply supported. Fiber volume ratio, 0.5; aspect ratio a/b, 2.

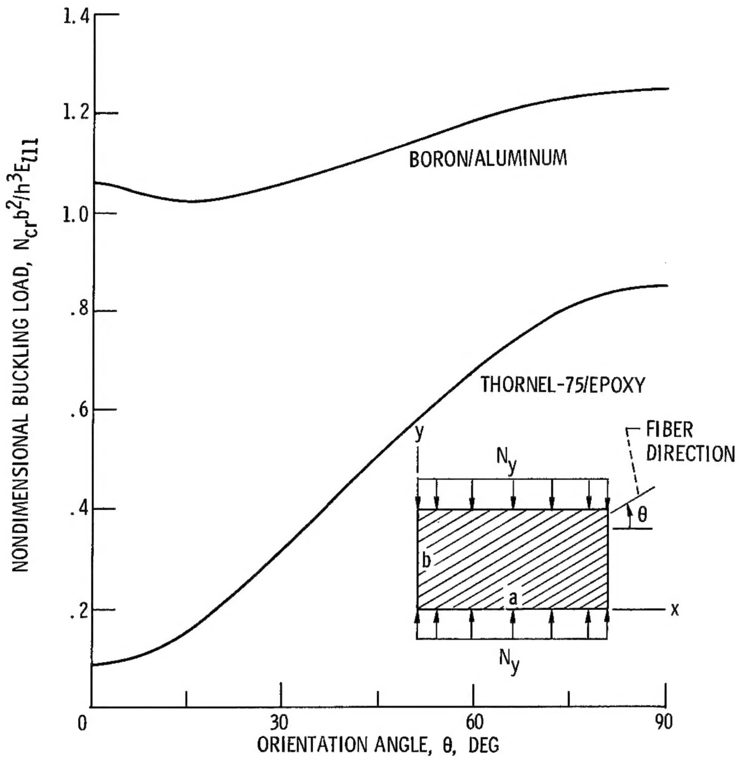


Figure 3. - Buckling loads for two fiber-composite plates - all four edges simply supported. Fiber volume ratio, 0.5; aspect ratio a/b, 2.

Thornel-75/resin composites in structures which are critical in buckling as measured by the specific buckling stress.

Buckling load comparisons where the panels are loaded in the y-direction are illustrated in figure 3. In this figure, it can be seen that the boron/aluminum composite panel is considerably stronger in buckling than the corresponding Thornel-75/epoxy panel. In this plot, the nondimensional buckling load parameter is plotted as a function of the orientation angle for the fixed panel aspect ratio of 2.

Buckling Loads for Individual Loading Conditions

Design data for boron/aluminum panels which are subjected to compressive load in the x-direction are illustrated in figure 4 as a function of orientation for various aspect ratios. The important point to be noted from this figure is that the buckling load is independent, or almost independent, of orientation angle in panels where the aspect ratio is approximately greater than 1. Another point to be noted is that the buckling load depends only moderately on the orientation angle in panels of aspect ratio less than 1.

A cross-plot of figure 4 is illustrated in figure 5. The nondimensional load is

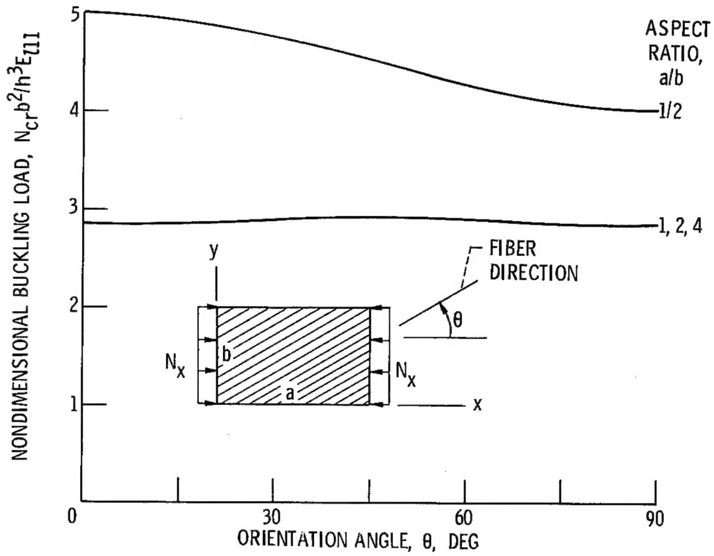


Figure 4. - Buckling loads for boron/aluminum composite plates, with all four edges simply supported, subjected to normal (N_x) compressive load. Fiber volume ratio, 0.5.

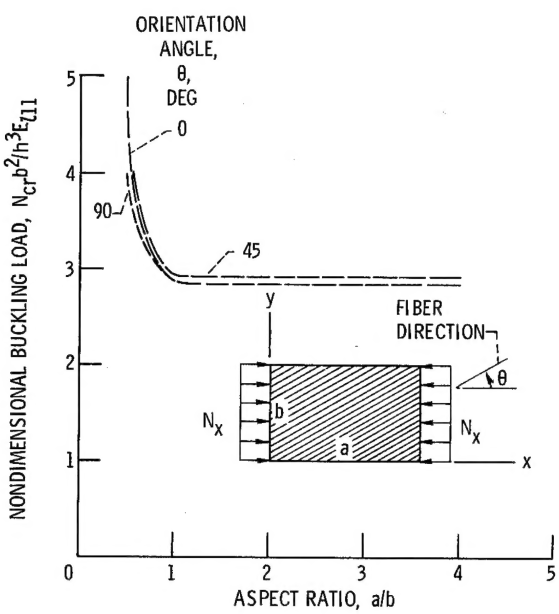


Figure 5. - Buckling loads for boron/aluminum composite plates, with all four edges simply supported, subjected to normal (N_x) compressive load. Fiber volume ratio, 0.5.

plotted as a function of panel aspect ratio for various orientation angles. The insensitivity of the buckling load as a function of orientation angle in panels with aspect ratios greater than about 1 is clearly illustrated in this figure. The dashed line symbol is used to represent these curves to emphasize that buckling load values were computed only at the aspect ratios 1/2, 1, 2, and 4.

Buckling loads for panels which are loaded in the y-direction only are given in figure 6. As can be seen in this figure, the buckling loads are almost independent of the orientation angle in panels with aspect ratios of 1/2 and greater. The buckling load, on the other hand, is very sensitive to the aspect ratio in panels with aspect ratios of approximately 2 or less.

Buckling loads for a boron/aluminum composite panel loaded with shear only are illustrated in figure 7. The points to be noted in this figure are the following:

(1) There is a mild buckling load dependence on the orientation angle for panel aspect ratios of less than about 1.

(2) The buckling load is relatively independent of orientation angle for panel aspect ratios of greater than about 1.

(3) The buckling load is very sensitive to the aspect ratio in panels with aspect ratios less than 2, and this dependence becomes rather insignificant for panel aspect ratios greater than 2.

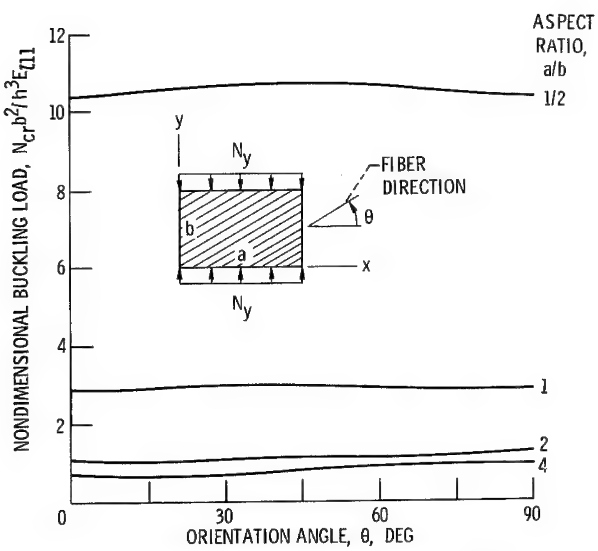


Figure 6. - Buckling loads for boron/aluminum composite plates, with all four edges simply supported, subjected to normal (N_y) compressive load. Fiber volume ratio, 0.5.

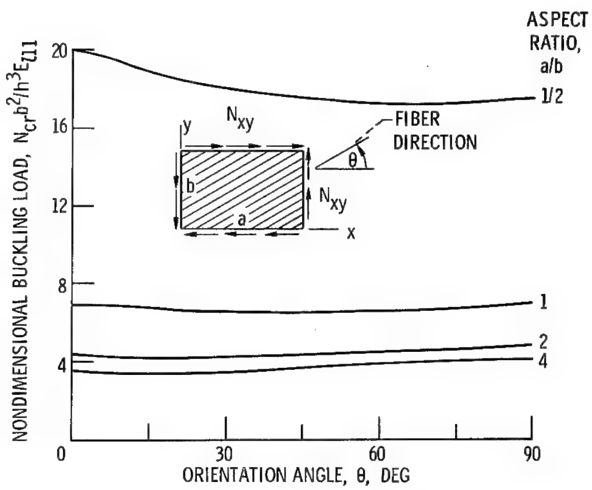


Figure 7. - Buckling loads for boron/aluminum composite plates, with all four edges simply supported, subjected to shear (N_{xy}) load. Fiber volume ratio, 0.5.

Buckling Loads for Two Equal Simultaneous Loadings

Buckling loads, when the panel is loaded with equal loads in the x- and y-directions, are shown in figure 8. In this figure, the nondimensional buckling load parameter is plotted as a function of orientation angle for various panel aspect ratios.

The results in this figure show that the buckling load is slightly dependent on the

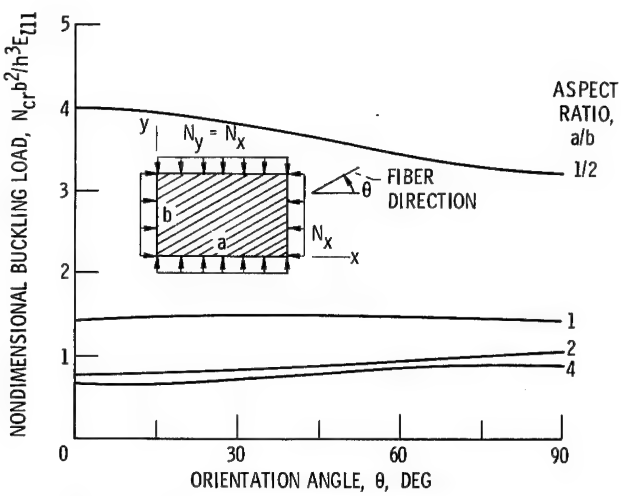


Figure 8. - Buckling loads for boron/aluminum composite plates, with all four edges simply supported, subjected to combined normal compressive ($N_x = N_y$) loads. Fiber volume ratio, 0.5.

orientation angle for panel aspect ratios less than 1, and practically independent of orientation angle for aspect ratios greater than 1. The buckling load is sensitive to panel aspect ratio for aspect ratios less than or equal to 2. This dependence becomes insignificant for panel aspect ratios greater than 2. The curves of the buckling load as a function of the independent variables indicated in figure 8 parallel the curves of the buckling loads indicated in figures 4 and 6, for the individual loadings.

Buckling loads for the case when the panel is loaded in the x-direction combined with shear are shown in figure 9. The curves of the buckling load for this loading condition are parallel to those of the individual cases (figs. 4 and 7). Buckling loads for the case when the panel is loaded in the y-direction combined with shear are illustrated in figure 10. The buckling load in this figure seems to be practically independent of orientation angle for the aspect ratios investigated. However, it is quite sensitive to the panel aspect ratio for aspect ratios less than 2.

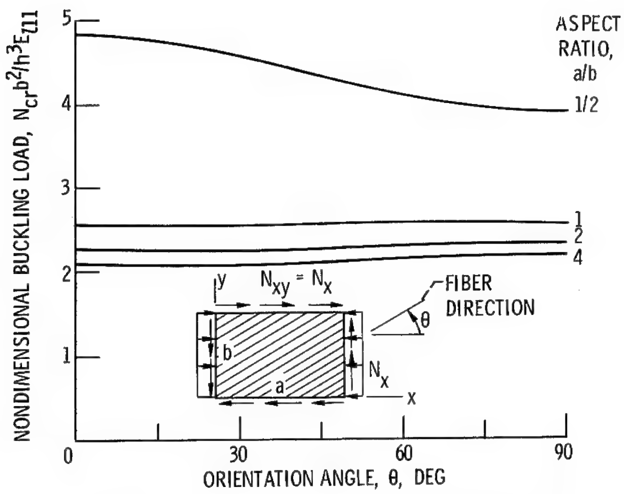


Figure 9. - Buckling loads for boron/aluminum composite plates, with all four edges simply supported, subjected to combined normal (N_x) and shear ($N_{xy} = N_x$) loads. Fiber volume ratio, 0.5.

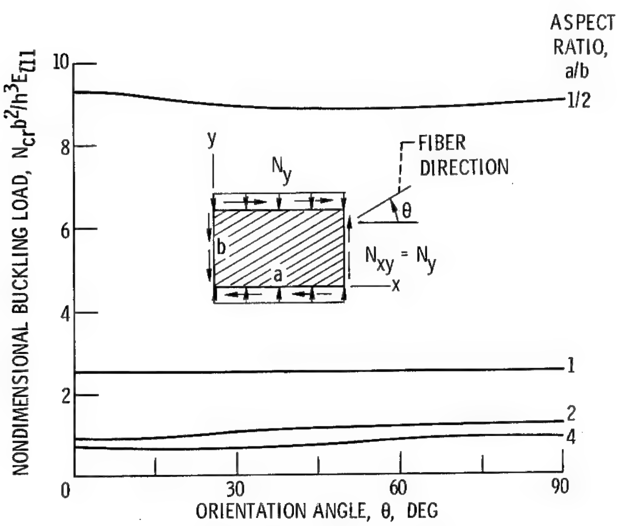


Figure 10. - Buckling loads for boron/aluminum composite plates, with all four edges simply supported, subjected to combined normal (N_y) and shear ($N_{xy} = N_y$) loads. Fiber volume ratio, 0.5.

Buckling Loads for Three Equal Simultaneous Loadings

Buckling loads for panels which are loaded in the x- and y-directions combined with shear are shown in figure 11. The schematic in this figure indicates the type of loadings and their respective ratios. The nondimensional buckling load is plotted as a function of orientation angle for various panel aspect ratios.

Comparing corresponding curves from figures 8 and 11, it is seen that the addition

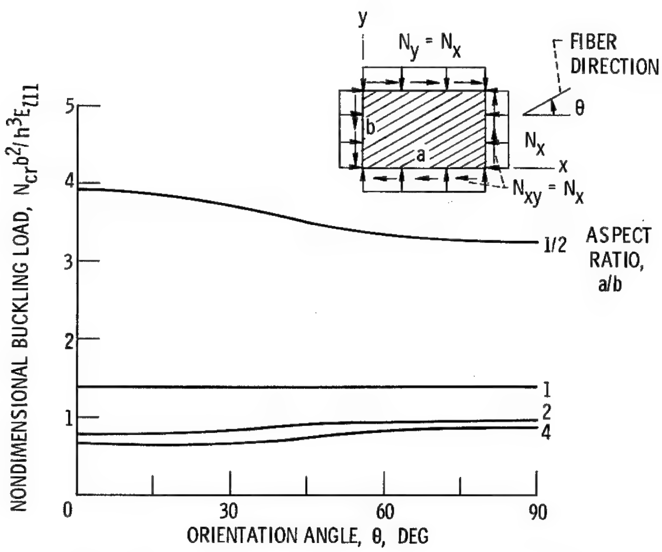


Figure 11. - Buckling loads for boron/aluminum composite plates, with all four edges simply supported, subjected to combined normal ($N_y = N_x$) and shear ($N_{xy} = N_x$) loads. Fiber volume ratio, 0.5.

of the shearing load decreases the buckling load of the panel only slightly. The point to be noted then is that a panel subjected to compressive loads in the x- and y-directions will resist almost an equal amount of shearing load for approximately the same buckling load.

Buckling Loads for Two or Three Unequal Simultaneous Loadings

Buckling loads for panels which are subjected to unequal loads in the x- and y-directions are shown in figure 12. The type of loading condition and respective loading magnitudes are illustrated in the sketch given in the figure. In this figure, the nondimensional buckling load parameter is plotted as a function of the orientation angle for various aspect ratios. The curves of the buckling load for this type of loading condition

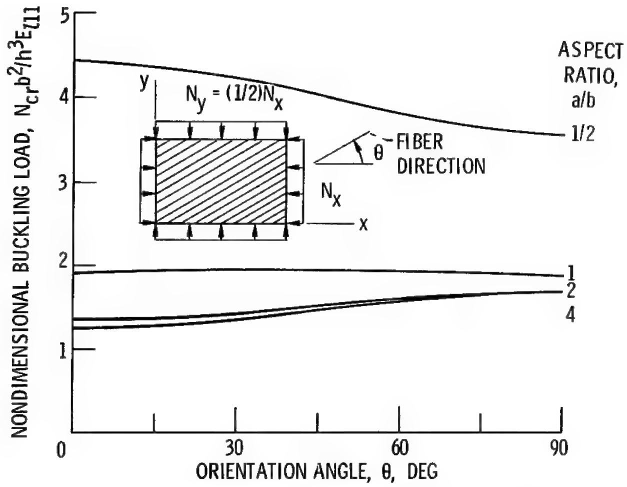


Figure 12. - Buckling loads for boron/aluminum composite plates, with all four edges simply supported, subjected to combined normal ($N_y = (1/2)N_x$) loads. Fiber volume ratio, 0.5.

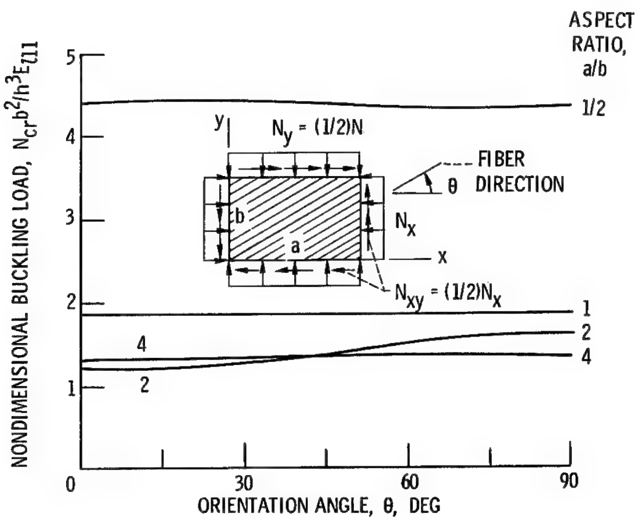


Figure 13. - Buckling loads for boron/aluminum composite plates, with all four edges simply supported, subjected to combined normal ($N_y = (1/2)N_x$) and shear ($N_{xy} = (1/2)N_x$) loads. Fiber volume ratio, 0.5.

parallel those of the cases with equal loading condition, as was previously discussed (see fig. 8). One additional point to be noted is that the buckling loads of panels with aspect ratios of greater than about 2 remain almost invariant as a function of aspect ratio when the orientation angle is greater than about 45°.

Buckling data for panels loaded with unequal combined loading conditions are illustrated in figure 13. The loading condition for the panel is illustrated in the schematic in the figure. The buckling load is plotted as a function of orientation angle for various

aspect ratios. A point to note in this figure is that, at some orientation angles, panels with aspect ratios greater than 2 could have greater buckling loads than panels with aspect ratios of 2. As can be seen, the buckling load for a panel with an aspect ratio of approximately 2 is lower than for the panel with an aspect ratio of 4 when the orientation angle is approximately less than 45^{0}

All the buckling data presented and discussed indicate that the buckling load is insensitive to orientation angle for panels with high aspect ratios. The buckling load is mildly sensitive to the orientation angle in panels with low aspect ratios. This observation leads to the important conclusion that the buckling loads of boron/aluminum composite anisotropic plates can be approximately determined by using classical orthotropic theory. This conclusion is indeed a useful one, since the buckling of orthotropic plates has been extensively treated in reference 5.

POSSIBLE EXTRAPOLATIONS OF DESIGN DATA

The design data presented and discussed were based on a fixed fiber volume ratio of 0.5. The data presented herein can be used to extrapolate buckling loads for plates made from composites with different fiber volume ratios. The results will be very close if the variation of the fiber volume ratio is within approximately ±20 percent of the 0.5 value which was used in generating the design data.

This extrapolation is justified since the buckling load is nondimensionalized with respect to both composite longitudinal modulus and thickness. It is well known that both composite modulus and thickness depend on the fiber volume ratio (ref. 10), and that this dependence is approximately linear in the fiber-volume-ratio range 0.4 to 0.6. In this sense, then, the extrapolation using the design data presented herein for fiber volume ratios within ±20 percent of 0.5 should yield reasonable results.

The computer code appended in appendix C can be slightly modified to compute the first natural frequency of anisotropic boron/aluminum composite panels. In reference 5, the vibration problem is discussed, and the analogy in computing the buckling load and the natural frequency is made.

CONCLUSIONS

The discussion of the theoretical design data presented leads to the following conclusions:

1. Design data for the buckling of unidirectional boron/aluminum panels with fibers oriented at any angle to the load direction have been generated and are reported herein.

2. Specific buckling stress comparisons showed that, in general, boron/aluminum composite panels are more efficient than high-modulus graphite/resin composite panels.

3. The buckling load of boron/aluminum unidirectional panels is practically independent of fiber direction at high aspect ratio values. At these aspect ratios, the plate can be assumed to have its material axes coincide with its structural axes. Consequently, the classical buckling theory of orthotropic plates can be used to predict the buckling load.

4. The buckling loads of boron/aluminum unidirectional panels are only moderately dependent on fiber direction at plate aspect ratios less than 1.

5. Boron/aluminum composite panels loaded by normal in-plane loads which are near the critical load can carry considerable shear load before they buckle.

6. The buckling loads of boron/aluminum panels are practically independent of aspect ratio at aspect ratios greater than about 3.

7. The buckling loads of panels with fiber volume ratios within approximately ±20 percent of 0.5 can be extrapolated from the design data presented herein by using the appropriate panel thickness and the appropriate composite longitudinal modulus.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, September 3, 1971,
129-03.

APPENDIX A

SYMBOLS

a	panel dimension, x-direction
b	panel dimension, y-direction
D	matrix of flexural rigidities, eq. (1)
h	thickness
K	plate stiffness matrix, elements given by eqs. (B1) and (B4)
L	load matrix, elements given by eqs. (B1) and (B4)
M	summation index limit on m
m	summation index
N	summation index limit on n; with subscripts, applied load
$\overline{N}$	parameter in eq. (B1)
N_{cr}	buckling load
n	summation index
p	summation index
q	summation index
W	vector of coefficients in the displacement mode expansion, eq. (1)
w	displacement in z-direction
x, y, z	panel structural axes
1, 2, 3	panel material axes
δ	variation
ϵ', ϵ''	convergence tolerances
θ	orientation angle of material axes with respect to structural axes
λ	buckling parameter
ρ	ratios defined by eq. (B5), also weight density
σ	stress and matrix of stresses

Subscripts:

cr	critical or buckling condition
i	row index

j	column index
l	undirectional composite property
m	buckling modes, x-direction, axes
n	buckling modes, y-direction
r	iteration cycle
x, y, z	directions associated with the respective structural axes
1, 2, 3	directions associated with the respective material axes

APPENDIX B

DESCRIPTION OF NUMERICAL ALGORITHM

The numerical algorithm seeks the eigenvalue of the following matrix equation:

$$[K]\{W\} = \bar{N}[L]\{W\} \tag{B1}$$

where [K] and [L] are $(M \times N) \times (M \times N)$ square matrices, {W} is a column matrix containing the W_{mn}, and $\bar{N}$ is defined subsequently. The (i, j) elements of the [K] and [L] matrices are given by the following indicial expressions:

$$\left.\begin{array}{ll} m = 1(1)M; & n = 1(1)N \\ p = 1(1)M; & q = 1(1)N \\ i = (m - 1)N + n; & j = (p - 1)N + q \end{array}\right\} \tag{B2}$$

$$\left.\begin{array}{l} \left.\begin{array}{l} K_{ii} = \dfrac{\pi^4}{4}\left[\dfrac{b}{a^3} m^4 D_{11} + \dfrac{2}{ab} m^2 n^2 (D_{12} + 2D_{33}) + \dfrac{a}{b^3} n^4 D_{22}\right] \\ L_{ii} = -\dfrac{\pi^2}{4}\left(\dfrac{b}{a} m^2 \rho_x + \dfrac{a}{b} n^2 \rho_y\right) \end{array}\right\} \quad p = m \text{ and } q = n \\ K_{ii} = L_{ii} = 0 \qquad p \neq m \text{ or } q \neq n \end{array}\right\} \tag{B3}$$

$$\left.\begin{array}{l} \left.\begin{array}{l} K_{ij} = -\dfrac{8\pi^2 mnpq}{a^2(q^2 - n^2)}\left(\dfrac{2m^2}{p^2 - m^2} + 1\right)D_{13} - \dfrac{8\pi^2 mnpq}{b^2(p^2 - m^2)}\left(\dfrac{2n^2}{q^2 - n^2} + 1\right)D_{23} \\ L_{ij} = -\dfrac{8mnpq}{(p^2 - m^2)(q^2 - n^2)}\rho_{xy} \end{array}\right\} \quad m + p \text{ and } n + q \text{ odd} \\ K_{ij} = L_{ij} = 0 \qquad m + p \text{ or } n + q \text{ even} \end{array}\right\} \tag{B4}$$

where

$$\rho_x = \frac{\overline{N}_x}{\overline{N}}; \qquad \rho_y = \frac{\overline{N}_y}{\overline{N}}; \qquad \rho_{xy} = \frac{\overline{N}_{xy}}{\overline{N}} \tag{B5}$$

where $\overline{N}$ is the buckling parameter. Here $\overline{N}_x$, $\overline{N}_y$, and $\overline{N}_{xy}$ carry their algebraic signs, whereas $\overline{N}$ is always taken as a positive quantity.

The buckling loads are determined by finding the largest eigenvalue in equation (B1) using the Power method. Several methods are available for finding eigenvalues of linear systems (ref. 3). The Power method is relatively easy to program and is applicable to nonsymmetric matrices which have real eigenvalues. The method yields the largest eigenvalue and the corresponding eigenvector. To apply the Power method, equation (B1) is expressed in the following form:

$$\lambda\{W\} = [K]^{-1}[L]\{W\} \tag{B6}$$

where $\lambda = 1/\overline{N}$. The solution is obtained in an iterative fashion as follows:

$$\lambda_{r+1}\{W\}_{r+1} = [K]^{-1}[L]\{W\}_r \tag{B7}$$

where $\{W\}_r$ is normalized relative to its largest element. Thus

$$\{W\}_r = \frac{\{W\}_r}{\lambda_r} \tag{B8}$$

where

$$\lambda_r = \max_i \left|\left(W_r\right)_i\right| \qquad (i = 1 \ \ (1) \ \ M \times N) \tag{B9}$$

The iteration process stops when

$$\frac{\lambda_{r+1} - \lambda_r}{\lambda_{r+1}} < \epsilon' \tag{B10}$$

where ϵ' is usually taken $10^{-4} \leq \epsilon' \leq 10^{-6}$. This procedure converges rapidly as long as the next largest eigenvalue is not close to λ. The Power method runs into difficul-

ties when the shear buckling load is sought because for this case the magnitude of the two largest eigenvalues is the same. This difficulty is easily overcome by using very small values for one or both of the normal loads in combination with the shear load as is described in reference 3.

The buckling load is obtained from the relation

$$\frac{1}{\overline{N}} = \lambda_{r+1}$$

and

$$\overline{N} = \frac{1}{\lambda_{r+1}} = N_{cr} \tag{B11}$$

Thus the smallest buckling load is obtained since λ_{r+1} is the largest eigenvalue of equation (B6).

The buckling load of the plate then is determined by incrementing M and N in equation (B1) and applying the Power method to compute $\overline{N}$ until the convergence criterion

$$\frac{\overline{N}(M + 1)(N + 1)}{\overline{N}(M \times N)} < \epsilon'' \tag{B12}$$

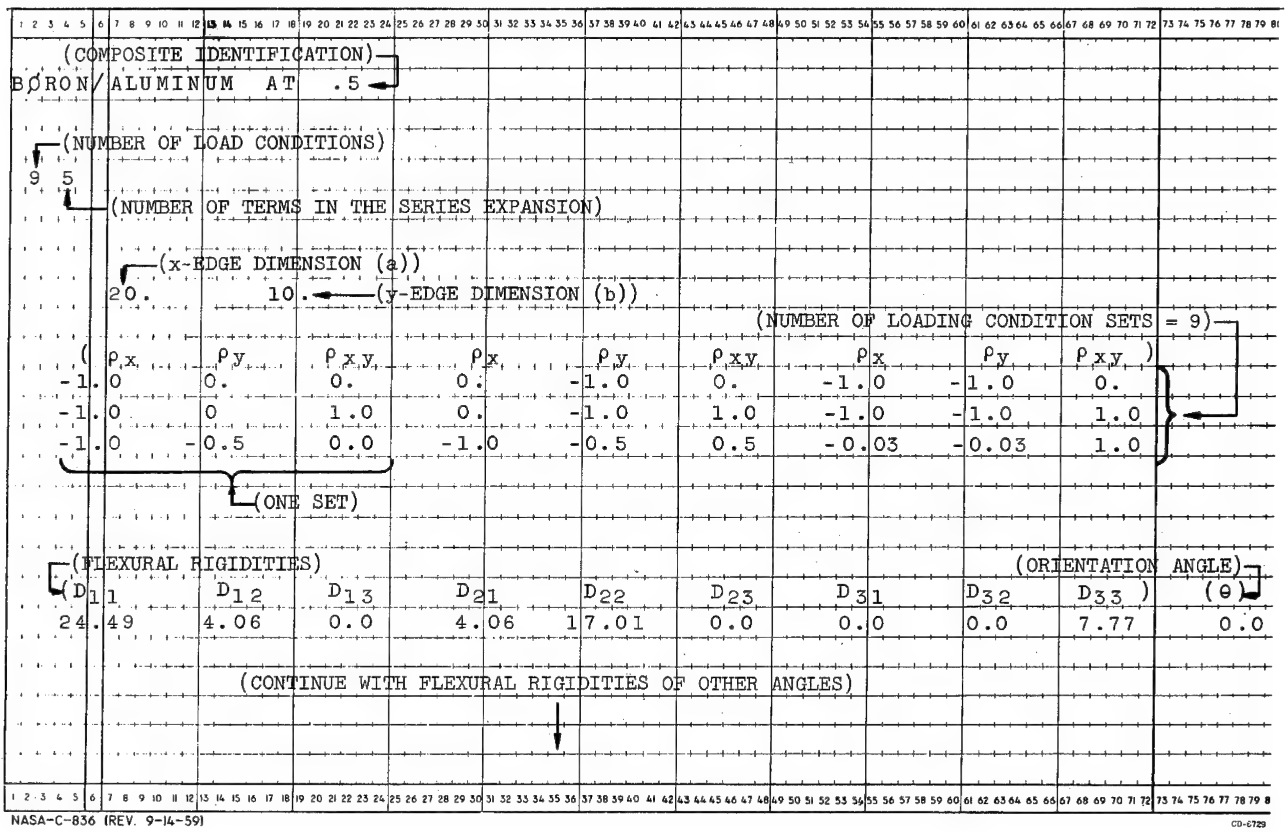
is satisfied. The parameter ϵ'' is usually chosen as an acceptable percentage of $\overline{N}$.

APPENDIX C

INPUT DATA

Explanations

Explanations appear in parentheses leadered to corresponding card.



Sample Input Data for 0.02-Inch-Thick Boron/Aluminum Plate

1-5	6	7-12	13-18	19-24	25-30	31-36	37-42	43-48	49-54	55-60	61-66	67-72	73-80
BØRØN	/	ALUMIN	UM AT	.5									
9 5													
		20.	10	.									
-1	.	0	0.	0.	0.	-	1.0	0.	-1.	0 -	1.0	0.	
-1	.	0	0.	1.0	0.	-	1.0	1.0	-1.	0 -	1.0	1.0	
-1	.	0 -	0.5	0.0	-1.	0 -	0.5	0.5	-0.	03 -	0.03	1.0	
24	.	49	4.06	0.0	4.	06 1	7.01	0.0	0.	0	0.0	7.77	0.0
24	.	42	4.08	.42	4.	08 1	7.05	.23	.	42	.23	7.79	5.0
20	.	17	4.64	1.87	4.	64 2	0.17	1.87	1.	87	1.87	8.34	45.0

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APPENDIX D

COMPUTER OUTPUT

```
C.... BUCKLING OF ANISOTROPIC PLATES
      COMMON /EIGVEC/ W(81)
      COMMON /ARFIL1/ D(3,3)/ARFIL2/RHO(3)/ARRAY/X(81,81)
      REAL N1,N2,LAMB2
      DIMENSION V(10,10), T(10), RST(20,3),TITLE(3)
      DATA PI/3.1415927/
      MODE=1
      READ(5,23) TITLE
      READ (5,12) NP,MU
      READ (5,13) A,B
      READ (5,14) ((RST(I,J),J=1,3),I=1,NP)
1     READ (5,25) ((D(I,J),J=1,3),I=1,3),THETA
      DO 11 II=1,NP
      WRITE (6,15)
      DO 2 J=1,3
2     RHO(J)=RST(II,J)
      N1=0.0
      M=1
      CALL TIMLFT (Q1)
3     MM=M*M
      CALL MFILL1 (M,A,B)
      CALL MINVA (MM,KSIG)
      IF (KSIG.EQ.2) GO TO 10
      CALL MFILL2 (M,A,B)
      CALL EIGFIN (MM,LAMB2)
      IF (KSIG.EQ.3) IFLAG=1
      N2=1.0/LAMB2
      EPS=ABS((N2-N1)/N2)
      IF (EPS.LE.1.0E-4) GO TO 5
      IF (M.EQ.MU) GO TO 4
      SPS=EPS
      M=M+1
      N1=N2
      GO TO 3
4     WRITE (6,16)
5     IF (IFLAG.EQ.1) WRITE (6,17)
      CALL TIMLFT (Q2)
      IQ=(Q1-Q2)/60.
      WRITE(6,24) TITLE,THETA
      WRITE (6,18) M,SPS,EPS,IQ
      VMAX=0.0
      DO 7 I=1,10
      DO 7 J=1,10
      V(I,J)=0.0
      DO 6 K=1,M
      DO 6 L=1,M
      IJ=(K-1)*M+L
6     V(I,J)=V(I,J)+W(IJ)*SIN(FLOAT(K*I)*PI/10.0)*SIN(FLOAT(L*J)*PI/10.)
7     VMAX=AMAX1(VMAX,ABS(V(I,J)))
      DO 8 I=1,10
      DO 8 J=1,10
```

```
8      V(I,J)=V(I,J)/VMAX
       WRITE (6,19) A,B,N2,MODE,RHO,D
       DO 9 I=1,10
       J=10-I+1
       T(I)=0.0
9      WRITE (6,20) J,T(I),(V(K,J),K=1,10)
       I=0
       XI=0.0
       WRITE (6,20) I,XI,(T(J),J=1,10)
       WRITE (6,21)
       GO TO 11
10     WRITE (6,22)
11     CONTINUE
       GO TO 1
C
12     FORMAT (2I2)
13     FORMAT (2F10.4)
14     FORMAT (9F8.3)
15     FORMAT (1H1)
16     FORMAT (23H MAXIMUM INDEX ATTAINED)
17     FORMAT (41H ILL-CONDITIONED MATRIX - RESULT IN DOUBT)
18     FORMAT (5X,2HM=,I1,5X,20HPREVIOUS REL. ERROR=,1PE10.3,5X,16HLAST R
      1EL. ERROR=,1PE10.3,5X,33H TIME REQUIRED FOR THIS CASE WAS ,I3,8H S
      2ECONDS)
19     FORMAT (///7X,1HA,13X,1HB,11X,3HNCR,11X,4HMODE,9X,5HRHO-X,9X,5HRHO
      1-Y,9X,6HRHO-XY//3(2X,1PE10.3,2X),6X,I1,7X,3(2X,1PE10.3,2X)///37H T
      2HE D-ARRAY COLUMNWISE IS AS FOLLOWS//9(2X,1PE10.3,2X)////50X,13HBU
      3CKLED SHAPE//6X,1HY)
20     FORMAT (//5X,I2,11(5X,F5.2))
21     FORMAT (//15X,1H0,9X,1H1,9X,1H2,9X,1H3,9X,1H4,9X,1H5,9X,1H6,9X,1H7
      1,9X,1H8,9X,1H9,8X,2H10,6X,1HX)
22     FORMAT (37H MATRIX SINGULAR - PROCESS TERMINATED)
23     FORMAT(3A6)
24     FORMAT(/20H COMPOSITE SYSTEM = ,3A6,12H AT THETA = ,F5.1)
25     FORMAT (10F8.3)
       END
```

```
      SUBROUTINE MFILL1 (MM,A,B)
      COMMON /ARFIL1/ D(3,3)
      COMMON /ARRAY/ X(81,81)
      INTEGER P,Q
      DATA PI/3.1415927/
      DO 3 M=1,MM
      DO 3 N=1,MM
      DO 3 P=1,MM
      DO 3 Q=1,MM
      I=(M-1)*MM+N
      J=(P-1)*MM+Q
      IF (I.NE.J) GO TO 1
      IF (P.NE.M.OR.Q.NE.N) GO TO 2
      X(I,J)=PI**4*(B*FLOAT(M**4)*D(1,1)/A**3+2.0*FLOAT(M*M*N*N)*(D(1,2)
     1+2.0*D(3,3))/(A*B)+A*FLOAT(N**4)*D(2,2)/B**3)/4.0
      GO TO 3
```

```
1     IF (MOD((M+P)*(N+Q),2).EQ.0) GO TO 2
      X(I,J)=-8.0*PI**2*D(1,3)*FLOAT(M*N*P*Q*(2*M*M/(P*P-M*M)+1)/(Q*Q-N*
     1N))/A**2-8.0*PI**2*D(2,3)*FLOAT(M*N*P*Q*(2*N*N/(Q*Q-N*N)+1)/(P*P-M
     2*M))/B**2
      GO TO 3
2     X(I,J)=0.0
3     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MINVA (M,KSIG)
C.....MATRIX INVERSION BY GAUSS-JORDAN ELIMINATION
      COMMON /ARRAY/ X(81,81)
      DIMENSION Y(81,81), Z(2,81,81), K(81)
      EQUIVALENCE (Y,Z)
      DOUBLE PRECISION A,R,PROD,Y
      DATA DELT,EPS,LOOPS/0.0001,1.0E-8,2/
      KSIG=0
      IF (M.NE.1) GO TO 3
      PROD=X(1,1)
      IF (PROD.NE.0.0) GO TO 2
1     KSIG=KSIG+2
      RETURN
2     X(1,1)=1.0/PROD
      RETURN
3     PROD=1.0
      MM=M-1
      DO 4 I=1,M
      K(I)=I
      DO 4 J=1,M
4     Y(I,J)=X(I,J)
C.....BEGIN BY FINDING LARGEST PIVOTAL ELEMENT
      DO 9 I=1,M
      A=0.0
      DO 5 J=I,M
      IF (ABS(Y(J,1)).LE.A) GO TO 5
      A=ABS(Y(J,1))
      L=J
5     CONTINUE
      IF (A.EQ.0.0) GO TO 1
C.....REARRANGE ROWS AND ORDER ARRAY
      N=K(L)
      K(L)=K(I)
      K(I)=N
      DO 6 J=1,M
      A=Y(I,J)
      Y(I,J)=Y(L,J)
6     Y(L,J)=A
C.....REDUCE PIVOTAL ROW
      A=Y(I,1)
      DO 7 J=1,MM
7     Y(I,J)=Y(I,J+1)/A
      Y(I,M)=1.0/A
```

```
C.....REDUCE REMAINING ROWS
      DO 9 L=1,M
      IF (L.EQ.I) GO TO 9
      A=Y(L,1)
      DO 8 N=1,MM
      Y(L,N)=Y(L,N+1)-A*Y(I,N)
8     IF (ABS(Y(L,N)).LT.(ABS(Y(L,N+1))*EPS)) Y(L,N)=0.0
      Y(L,M)=-A*Y(I,M)
9     CONTINUE
C.....UNSCRAMBLE INVERTED MATRIX
      DO 13 I=1,M
      IF (K(I).EQ.I) GO TO 13
      DO 10 J=I,M
10    IF (K(J).EQ.I) GO TO 11
      GO TO 1
11    DO 12 L=1,M
      A=Y(L,I)
      Y(L,I)=Y(L,J)
12    Y(L,J)=A
      K(J)=K(I)
13    CONTINUE
C.....OBTAIN ERROR MATRIX
      DO 18 N=1,LOOPS
      TEST=0.0
      DO 15 I=1,M
      DO 15 J=1,M
      R=0.0
      DO 14 L=1,M
14    R=R-Z(1,L,J)*X(I,L)
      IF (I.EQ.J) R=R+1.0
      TEST=AMAX1(TEST,ABS(R))
15    Z(2,I,J)=R
      DO 17 I=1,M
      DO 17 J=1,M
      A=0.0
      DO 16 L=1,M
16    A=A+Z(1,I,L)*Z(2,L,J)
17    Z(1,I,J)=Z(1,I,J)+A
      IF (TEST.LE.DELT) GO TO 19
18    CONTINUE
      KSIG=KSIG+3
C.....TRANSFER FINAL INVERSE
19    DO 20 I=1,M
      DO 20 J=1,M
20    X(I,J)=Z(1,I,J)
      RETURN
      END



      SUBROUTINE MFILL2 (MM,A,B)
      COMMON /ARFIL2/ RHO(3)
      COMMON /ARRAY/ X(81,81)
      DIMENSION L(81,81), C(81,81)
      REAL L
      INTEGER P,C
      DATA PI/3.1415927/
```

```
      DO 3 M=1,MM
      DO 3 N=1,MM
      DO 3 P=1,MM
      DO 3 Q=1,MM
      I=(M-1)*MM+N
      J=(P-1)*MM+Q
      IF (I.NE.J) GO TO 1
      IF (P.NE.M.OR.Q.NE.N) GO TO 2
      L(I,J)=-PI**2*(B*FLOAT(M*M)*RHO(1)/A+A*FLOAT(N*N)*RHO(2)/B)/4.0
      GO TO 3
1     IF (MOD((M+P)*(N+Q),2).EQ.0) GO TO 2
      L(I,J)=-RHO(3)*FLOAT(8*M*N*P*Q/((P*P-M*M)*(Q*Q-N*N)))
      GO TO 3
2     L(I,J)=0.0
3     CONTINUE
      NN=MM*MM
      DO 5 I=1,NN
      DO 5 J=1,NN
      SUM=0.0
      DO 4 IJ=1,NN
4     SUM=SUM+X(I,IJ)*L(IJ,J)
5     C(I,J)=SUM
      DO 6 I=1,NN
      DO 6 J=1,NN
6     X(I,J)=C(I,J)
      RETURN
      END
```

```
      SUBROUTINE EIGFIN (MM,LAMB2)
      REAL LAMB1,LAMB2
      COMMON /EIGVEC/ W(81)
      COMMON /ARRAY/ X(81,81)
      DIMENSION V(81)
      LAMB1=0.0
      W(1)=0.5
      IF (MM.EQ.1) GO TO 2
      DO 1 I=2,MM
1     W(I)=0.5
2     DO 4 I=1,MM
      SUM=0.0
      DO 3 J=1,MM
3     SUM=SUM+X(I,J)*W(J)
4     V(I)=SUM
      LAMB2=ABS(V(1))
      IF (MM.EQ.1) GO TO 6
      DO 5 I=2,MM
5     LAMB2=AMAX1(LAMB2,ABS(V(I)))
6     DO 7 I=1,MM
7     W(I)=V(I)/LAMB2
      IF (ABS((LAMB2-LAMB1)/LAMB2).LE.1.0E-4) RETURN
      LAMB1=LAMB2
      GO TO 2
      END
```

MAXIMUM INDEX ATTAINED

COMPOSITE SYSTEM = BORON/ALUMINUM AT THETA = 30.0
N=5 PREVIOUS REL. ERROR= 1.554E-04 LAST REL. ERROR= 3.944E-04 TIME REQUIRED FOR THIS CASE WAS 11 SECONDS

A	B	NCR	MODE	RHO-X	RHO-Y	RHO-XY
2.000E+01	1.000E+01	8.111E+00	1	-1.000E+00	0.	0.

THE C-ARRAY COLUMNWISE IS AS FOLLOWS

2.219E+01 4.490E+00 1.870E+00 4.490E+00 1.845E+01 1.370E+00 1.870E+00 1.370E+00 8.200E+00

BUCKLED SHAPE

Y												
10	0.	-0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	0.00	-0.00	
9	0.	0.20	0.31	0.26	0.12	-0.04	-0.14	-0.18	-0.15	-0.09	0.00	
8	0.	0.39	0.58	0.50	0.22	-0.08	-0.27	-0.34	-0.28	-0.16	0.00	
7	0.	0.55	0.80	0.67	0.29	-0.11	-0.38	-0.46	-0.39	-0.22	0.00	
6	0.	0.65	0.95	0.79	0.33	-0.14	-0.45	-0.54	-0.45	-0.25	0.00	
5	0.	0.69	1.00	0.82	0.34	-0.16	-0.47	-0.56	-0.47	-0.26	0.00	
4	0.	0.66	0.95	0.77	0.31	-0.16	-0.45	-0.54	-0.44	-0.24	0.00	
3	0.	0.57	0.81	0.65	0.25	-0.14	-0.39	-0.45	-0.37	-0.20	0.00	
2	0.	0.42	0.59	0.46	0.18	-0.11	-0.28	-0.33	-0.27	-0.14	0.00	
1	0.	0.22	0.31	0.24	0.09	-0.06	-0.15	-0.17	-0.14	-0.07	0.00	
0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
	0	1	2	3	4	5	6	7	8	9	10	X

```
COMPOSITE SYSTEM = BORON/ALUMINUM      AT THETA =  30.0
    N=3      PREVIOUS REL. ERROR= 8.298E-04      LAST REL. ERROR= 3.563E-06       TIME REQUIRED FOR THIS CASE WAS    5 SECONDS
```

A	B	NCR	MODE	RHO-X	RHO-Y	RHO-XY
2.000E+01	1.000E+01	2.986E+00	1	0.	-1.000E+00	0.

THE C-ARRAY COLUMNWISE IS AS FOLLOWS

2.219E+01	4.490E+00	1.870E+00	4.490E+00	1.845E+01	1.370E+00	1.870E+00	1.370E+00	8.200E+00

BUCKLED SHAPE

Y											
10	0.	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00
9	0.	0.09	0.18	0.24	0.29	0.31	0.30	0.25	0.18	0.10	-0.00
8	0.	0.18	0.34	0.47	0.55	0.59	0.56	0.48	0.35	0.18	-0.00
7	0.	0.25	0.47	0.65	0.76	0.81	0.77	0.66	0.48	0.25	-0.00
6	0.	0.29	0.55	0.76	0.90	0.95	0.90	0.77	0.56	0.29	-0.00
5	0.	0.31	0.59	0.81	0.95	1.00	0.95	0.81	0.58	0.31	-0.00
4	0.	0.30	0.57	0.78	0.91	0.95	0.90	0.76	0.55	0.29	-0.00
3	0.	0.26	0.49	0.66	0.78	0.81	0.77	0.65	0.47	0.24	-0.00
2	0.	0.19	0.36	0.49	0.57	0.59	0.55	0.47	0.34	0.18	-0.00
1	0.	0.10	0.19	0.26	0.30	0.31	0.29	0.25	0.18	0.09	-0.00
0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0	1	2	3	4	5	6	7	8	9	10 X

```
COMPCSITE SYSTEM = BORON/ALUMINUM      AT THETA =  30.0
   N=3     PREVIOUS REL. ERROR= 8.298E-04     LAST REL. ERROR= 5.726E-06     TIME REQUIRED FOR THIS CASE WAS   6 SECONDS
```

A	B	NCR	MODE	RHO-X	RHO-Y	RHO-XY
2.CC0E+01	1.000E+01	2.389E+00	1	-1.000E+00	-1.000E+00	0.

THE C-ARRAY CCLUMNWISE IS AS FOLLOWS

2.219E+01	4.490E+00	1.870E+00	4.490E+00	1.845E+01	1.370E+00	1.870E+00	1.370E+00	8.200E+00

BUCKLED SHAPE

Y												
10	0.	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	
9	0.	0.10	0.19	0.26	0.30	0.31	0.29	0.24	0.17	0.09	-0.00	
8	0.	0.19	0.36	0.49	0.57	0.59	0.55	0.46	0.33	0.17	-0.00	
7	0.	0.27	0.50	0.68	0.78	0.81	0.75	0.63	0.45	0.24	-0.00	
6	0.	0.31	0.59	0.80	0.93	0.95	0.88	0.74	0.53	0.27	-0.00	
5	0.	0.33	0.63	0.85	0.98	1.00	0.93	0.77	0.55	0.29	-0.00	
4	0.	0.32	0.60	0.81	0.93	0.95	0.88	0.73	0.52	0.27	-0.00	
3	0.	0.28	0.52	0.69	0.79	0.81	0.74	0.62	0.44	0.23	-0.00	
2	0.	0.20	0.38	0.51	0.58	0.59	0.54	0.45	0.32	0.16	-0.00	
1	0.	0.11	0.20	0.27	0.30	0.31	0.28	0.23	0.17	0.09	-0.00	
C	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
	C	1	2	3	4	5	6	7	8	9	10	X

MAXIMUM INDEX ATTAINED

COMPOSITE SYSTEM = BORON/ALUMINUM AT THETA = 30.0
N=5 PREVIOUS REL. ERROR= 1.910E-02 LAST REL. ERROR= 1.710E-03 TIME REQUIRED FOR THIS CASE WAS 11 SECONDS

A	B	NCR	MODE	RHO-X	RHO-Y	RHO-XY
2.000E+01	1.000E+01	6.185E+00	1	-1.000E+00	0.	-1.000E+00

THE C-ARRAY COLUMNWISE IS AS FOLLOWS

2.219E+01	4.490E+00	1.870E+00	4.490E+00	1.845E+01	1.370E+00	1.870E+00	1.370E+00	8.200E+00

BUCKLED SHAPE

Y											
10	0.	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	-0.00
9	0.	0.10	0.22	0.32	0.30	0.16	-0.06	-0.24	-0.31	-0.22	0.00
8	0.	0.21	0.45	0.61	0.55	0.25	-0.17	-0.50	-0.58	-0.39	0.00
7	0.	0.34	0.66	0.84	0.70	0.23	-0.34	-0.73	-0.79	-0.50	0.00
6	0.	0.45	0.83	0.97	0.72	0.12	-0.53	-0.91	-0.89	-0.53	0.00
5	0.	0.53	0.92	0.99	0.63	-0.03	-0.68	-1.00	-0.90	-0.51	0.00
4	0.	0.55	0.91	0.90	0.47	-0.19	-0.76	-0.97	-0.81	-0.43	0.00
3	0.	0.51	0.80	0.71	0.29	-0.29	-0.72	-0.83	-0.64	-0.32	0.00
2	0.	0.40	0.59	0.47	0.13	-0.29	-0.57	-0.60	-0.43	-0.20	0.00
1	0.	0.22	0.31	0.23	0.03	-0.18	-0.31	-0.31	-0.21	-0.09	0.00
0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0	1	2	3	4	5	6	7	8	9	10 X

MAXIMUM INDEX ATTAINED

COMPOSITE SYSTEM = BORON/ALUMINUM AT THETA = 30.0
N=5 PREVIOUS REL. ERROR= 1.603E-03 LAST REL. ERROR= 2.138E-04 TIME REQUIRED FOR THIS CASE WAS 12 SECONDS

A	B	NCR	MODE	RHO-X	RHO-Y	RHO-XY
2.000E+01	1.000E+01	2.863E+00	1	0.	-1.000E+00	-1.000E+00

THE C-ARRAY COLUMNWISE IS AS FOLLOWS

2.219E+01	4.490E+00	1.870E+00	4.490E+00	1.845E+01	1.370E+00	1.870E+00	1.370E+00	8.200E+00

BUCKLED SHAPE

Y											
10	0.	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00
9	0.	0.07	0.14	0.21	0.27	0.31	0.31	0.28	0.21	0.12	-0.00
8	0.	0.14	0.28	0.42	0.53	0.58	0.58	0.52	0.39	0.21	-0.00
7	0.	0.20	0.41	0.60	0.74	0.81	0.79	0.69	0.51	0.27	-0.00
6	0.	0.26	0.51	0.73	0.89	0.95	0.92	0.79	0.57	0.30	-0.00
5	0.	0.29	0.57	0.80	0.95	1.00	0.95	0.80	0.57	0.29	-0.00
4	0.	0.30	0.57	0.79	0.92	0.95	0.88	0.73	0.51	0.26	-0.00
3	0.	0.28	0.52	0.69	0.79	0.80	0.74	0.59	0.40	0.20	-0.00
2	0.	0.21	0.39	0.52	0.58	0.58	0.52	0.41	0.28	0.14	-0.00
1	0.	0.12	0.21	0.28	0.31	0.30	0.27	0.21	0.14	0.07	-0.00
0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0	1	2	3	4	5	6	7	8	9	10 X

MAXIMUM INDEX ATTAINED

COMPOSITE SYSTEM = BORON/ALUMINUM AT THETA = 30.0
M=5 PREVIOUS REL. ERROR= 1.267E-03 LAST REL. ERROR= 1.331E-04 TIME REQUIRED FOR THIS CASE WAS 11 SECONDS

A	B	NCR	MODE	RHO-X	RHO-Y	RHO-XY
2.000E+01	1.000E+01	2.320E+00	1	-1.000E+00	-1.000E+00	-1.000E+00

THE C-ARRAY COLUMNWISE IS AS FOLLOWS

2.219E+01	4.490E+00	1.870E+00	4.490E+00	1.845E+01	1.370E+00	1.870E+00	1.370E+00	8.200E+00

BUCKLED SHAPE

Y											
10	0.	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00
9	0.	0.07	0.15	0.22	0.28	0.31	0.30	0.27	0.20	0.11	-0.00
8	0.	0.15	0.29	0.43	0.54	0.59	0.57	0.50	0.38	0.20	-0.00
7	0.	0.21	0.43	0.62	0.75	0.81	0.78	0.68	0.50	0.26	-0.00
6	0.	0.27	0.53	0.75	0.90	0.95	0.91	0.77	0.56	0.29	-0.00
5	0.	0.30	0.58	0.81	0.96	1.00	0.94	0.79	0.56	0.29	-0.00
4	0.	0.31	0.58	0.79	0.92	0.95	0.88	0.72	0.50	0.26	-0.00
3	0.	0.28	0.52	0.69	0.79	0.80	0.73	0.59	0.41	0.20	-0.00
2	0.	0.21	0.39	0.52	0.58	0.58	0.52	0.42	0.28	0.14	-0.00
1	0.	0.12	0.21	0.27	0.31	0.30	0.27	0.21	0.14	0.07	-0.00
0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0	1	2	3	4	5	6	7	8	9	10 X

MAXIMUM INDEX ATTAINED

COMPOSITE SYSTEM = BORON/ALUMINUM AT THETA = 30.0
N=5 PREVIOUS REL. ERROR= 1.302E-03 LAST REL. ERROR= 2.519E-03 TIME REQUIRED FOR THIS CASE WAS 11 SECONDS

A	B	NCR	MODE	RHO-X	RHO-Y	RHO-XY
2.000E+01	1.000E+01	3.896E+00	1	-1.000E+00	-5.000E-01	-5.000E-01

THE C-ARRAY COLUMNWISE IS AS FOLLOWS

2.219E+01 4.490E+00 1.870E+00 4.490E+00 1.845E+01 1.370E+00 1.870E+00 1.370E+00 8.200E+00

BUCKLED SHAPE

Y											
10	0.	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00
9	0.	0.08	0.16	0.24	0.29	0.31	0.30	0.26	0.19	0.10	-0.00
8	0.	0.16	0.32	0.46	0.55	0.59	0.57	0.48	0.36	0.19	-0.00
7	0.	0.23	0.45	0.64	0.77	0.81	0.77	0.65	0.47	0.25	-0.00
6	0.	0.29	0.56	0.78	0.92	0.95	0.89	0.75	0.53	0.28	-0.00
5	0.	0.32	0.61	0.84	0.97	1.00	0.92	0.76	0.54	0.27	-0.00
4	0.	0.32	0.60	0.82	0.93	0.95	0.87	0.70	0.49	0.25	-0.00
3	0.	0.29	0.53	0.71	0.80	0.80	0.72	0.58	0.39	0.20	-0.00
2	0.	0.22	0.40	0.52	0.58	0.58	0.52	0.41	0.27	0.14	-0.00
1	0.	0.12	0.22	0.28	0.31	0.30	0.27	0.21	0.14	0.07	-0.00
0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0	1	2	3	4	5	6	7	8	9	10 X

COMPOSITE SYSTEM = BORON/ALUMINUM AT THETA = 30.0
N=3 PREVIOUS REL. ERROR= 8.298E-04 LAST REL. ERROR= 5.188E-05 TIME REQUIRED FOR THIS CASE WAS 15 SECONDS

A	B	NCR	MODE	RHO-X	RHO-Y	RHO-XY
2.000E+01	1.000E+01	3.982E+00	1	-1.000E+00	-5.000E-01	-0.

THE D-ARRAY COLUMNWISE IS AS FOLLOWS

2.219E+01 4.490E+00 1.870E+00 4.490E+00 1.845E+01 1.370E+00 1.870E+00 1.370E+00 8.200E+00

BUCKLED SHAPE

Y											
10	0.	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00
9	0.	0.12	0.22	0.28	0.31	0.30	0.26	0.20	0.14	0.07	-0.00
8	0.	0.23	0.42	0.54	0.59	0.56	0.49	0.38	0.26	0.13	-0.00
7	0.	0.32	0.58	0.75	0.81	0.77	0.67	0.52	0.35	0.18	-0.00
6	0.	0.37	0.68	0.88	0.95	0.91	0.78	0.61	0.41	0.21	-0.00
5	0.	0.40	0.72	0.93	1.00	0.95	0.82	0.64	0.43	0.22	-0.00
4	0.	0.38	0.69	0.89	0.95	0.90	0.77	0.60	0.41	0.21	-0.00
3	0.	0.33	0.59	0.76	0.81	0.77	0.66	0.51	0.34	0.17	-0.00
2	0.	0.24	0.43	0.55	0.59	0.55	0.47	0.37	0.25	0.13	-0.00
1	0.	0.13	0.23	0.29	0.31	0.29	0.25	0.19	0.13	0.07	-0.00
0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0	1	2	3	4	5	6	7	8	9	10 X

MAXIMUM INDEX ATTAINED

COMPOSITE SYSTEM = BORON/ALUMINUM AT THETA = 30.0
M=5 PREVIOUS REL. ERROR= 3.292E-02 LAST REL. ERROR= 2.308E-03 TIME REQUIRED FOR THIS CASE WAS 11 SECONDS

A	B	NCR	MODE	RHO-X	RHO-Y	RHO-XY
2.000E+01	1.000E+01	1.206E+01	1	-4.000E-02	-4.000E-02	-1.000E+00

THE C-ARRAY COLUMNWISE IS AS FOLLOWS

2.219E+01	4.490E+00	1.870E+00	4.490E+00	1.845E+01	1.370E+00	1.870E+00	1.370E+00	8.200E+00

BUCKLED SHAPE

Y											
10	0.	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00
9	0.	-0.01	0.01	0.09	0.22	0.32	0.32	0.21	0.08	0.00	-0.00
8	0.	-0.02	0.05	0.24	0.47	0.61	0.56	0.33	0.09	-0.02	-0.00
7	0.	0.01	0.15	0.43	0.72	0.83	0.67	0.34	0.04	-0.06	-0.00
6	0.	0.07	0.29	0.64	0.92	0.93	0.64	0.24	-0.05	-0.10	-0.00
5	0.	0.15	0.44	0.80	1.00	0.88	0.49	0.09	-0.14	-0.14	-0.00
4	0.	0.23	0.54	0.86	0.94	0.71	0.29	-0.06	-0.19	-0.14	0.00
3	0.	0.27	0.57	0.78	0.76	0.48	0.10	-0.14	-0.19	-0.11	0.00
2	0.	0.25	0.48	0.59	0.50	0.25	-0.01	-0.15	-0.15	-0.07	0.00
1	0.	0.15	0.27	0.31	0.24	0.10	-0.04	-0.10	-0.08	-0.03	0.00
0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
	0	1	2	3	4	5	6	7	8	9	10 X

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